

Math 614 Take-Home Assignment Key

1) $f(x) = \frac{4}{x}$

(A) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h} = \lim_{h \rightarrow 0} \frac{4x - 4(x+h)}{xh(x+h)}$
 $= \lim_{h \rightarrow 0} \frac{4x - 4x - 4h}{xh(x+h)} = \lim_{h \rightarrow 0} \frac{-4h}{xh(x+h)} = \lim_{h \rightarrow 0} \frac{-4}{x(x+h)}$
 $= \frac{-4}{x(x+0)} = \frac{-4}{x^2} \quad \therefore \boxed{f'(x) = -\frac{4}{x^2}}$

(B) slope of tangent line to the graph of f at $(-2, -2)$: $f'(-2) = -\frac{4}{(-2)^2} = -\frac{4}{4} = \boxed{-1}$

(C) equation of the tangent line in part (B):
 $y - y_1 = m(x - x_1)$
 $y + 2 = -1(x + 2)$
 $y + 2 = -x - 2$
 $\boxed{x + y = -4}$

