

## Math 205 Section 4.2 Greatest Common Divisor and Least Common Multiple

### PRIME FACTORIZATION

Composite numbers can always be written as a product of primes. Such a product is called the **prime factorization** of a number. For this reason, prime numbers are often referred to as the *building blocks of the whole numbers*.

**Example 1:** Find the prime factorization of each number. (A) 152 (B) 200 (C) 429

**Example 2:** List all the factors of each number. (A) 40 (B) 144 (C) 125

### GREATEST COMMON FACTOR (DIVISOR) AND LEAST COMMON MULTIPLE

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of a number. For this reason, prime numbers are often referred to as the *building blocks of the whole numbers*.

### FUNDAMENTAL THEOREM OF ARITHMETIC

Every composite whole number can be expressed as the product of primes in exactly one way (the order of the factors is disregarded).

**Example 3:** The product of the ages of a group of teenagers is 10,584,000. Find the number of teenagers in the group

and their ages.

### Greatest Common Factor

For any two numbers, there is always a number that is a factor of both. When a number is a factor of two numbers,

it is called a **common factor** or **common divisor**. Among the common factors of two numbers, there will always

be a largest number, which is called the **greatest common factor**.

For any two nonzero whole numbers  $a$  and  $b$ , the **greatest common factor**, written  $\text{GCF}(a, b)$ , is the greatest factor

(divisor) of both  $a$  and  $b$ .

Two numbers whose GCF is 1 are **relatively prime**.

**Example 4:** Find the greatest common factors. A)  $\text{GCF}(180, 220)$  B)  $\text{GCF}(96, 136, 220)$

### Least Common Multiple

A number is called a **common multiple** of two numbers if it is a multiple of both. Among these common multiples

will always be a smallest number, which is called the **least common multiple**.

For any two nonzero whole numbers  $a$  and  $b$ , the **least common multiple**, written  $\text{LCM}(a, b)$ , is the smallest multiple of both  $a$  and  $b$ .

**Example 5:** Find the least common multiples. A)  $a = 40, b = 66$  B)  $a = 15, b = 36, c = 55$

### Problem Solving Application

**Example 6:** The U.S. Census clock described at the beginning of this section used flashing lights to indicate birth, death, immigration, and emigration rates. The birth, death, immigration, and emigration lights flashed every 10, 16, 81, and 900 seconds, respectively.

A) Which of the other lights flashed every 900 seconds?

B) If all four lights flashed at the same moment, how much time would pass before they would all flash together again?

C) How long is the answer in (B) in hours?

D) What would be the total gain in population for each of the time intervals found in (B)?