Problem 3: Sketch sets of chips to illustrate and compute each of the following differences.

(A) $2 - 3$ (B) $4 - 2$ (C) $10 - (-5)$

Multiplication

The familiar rules for multiplying with negative numbers, such as "a negative times a negative is a positive," are easy enough to remember but difficult to illustrate.

Black and Red Chip Model - Multiplication by a positive integer can be illustrated by putting in groups of chips. Suppose you have $6 and you incur 2 debts, each for $3. These are represented by 6 black chips and 2 groups of 3 red chips. Now suppose that these 2 debts are removed. Removing the 2 groups of 3 red chips illustrates the product -2×-3. Notice that removing 6 red chips changes the value of the set from zero to 6. This suggests that $-2 \times -3 = 6$.

Problem 4: Sketch sets of chips to illustrate each computation. Describe the product in terms of debts and credits.

(A) $2 \times 3 = 6$ (B) $3 \times (-4) = -12$ (C) $(-3) \times (-2) = 6$

Rules of Signs for Multiplication

Let **a** and **b** be positive integers:

1. Positive times negative equals negative.

   $\mathbf{a} \times (-\mathbf{b}) = -(\mathbf{a} \times \mathbf{b})$

2. Negative times positive equals negative.

   $(-\mathbf{a}) \times \mathbf{b} = -(\mathbf{a} \times \mathbf{b})$

3. Negative times negative equals positive.

   $(-\mathbf{a}) \times (-\mathbf{b}) = \mathbf{a} \times \mathbf{b}$

Division

Both the *sharing* (*partitive*) and *measurement* (*subtractive*) concepts of division will be used in the following illustrations of division with the black and red chips model.

To show $-8 \div (-2)$, we begin with 8 red chips and then measure off, or subtract, as many groups of 2 red chips as possible. Since there are 4 such groups, $-8 \div (-2) = 4$. This illustration uses the *measurement* concept of division.

To show $-6 \div 3$, we divide 6 red chips into 3 equal groups. Since there are 2 red chips in each group, $-6 \div 3 = (-2)$. In this illustration, the divisor 3 indicates the number of equal parts into which the set is divided. This illustration uses the *sharing* concept of division.

Problem 5: Sketch sets of chips to illustrate each computation. Identify which division concept you illustrate.

(A) $(-24) \div (-4)$ (B) $(-30) \div 6$ (C) $18 \div (-3)$

Models for Integers

Black and Red Chips Model

The red and black rods used by the Chinese for positive and negative integers suggest a physical model for the integers. In place of rods we will use chips, and the color scheme will be reversed; that is, black chips will represent positive integers, and red chips negative integers. By establishing that each black chip together with a red chip represents 0 (think of each black chip as a $1 credit and each red chip as a $1 debt), we can represent every integer in an infinite number of ways.

Problem 1: Describe or illustrate four different sets of chips that represent –3.

Addition - Addition of integers can be illustrated by putting together (taking the union of) sets of black and red chips.

Problem 2: Sketch sets of chips to illustrate and compute each of the following sums.

(A) –4 + 2 (B) –2 + –1 (C) –1 + 5 (D) 2 + 3

Rules of Signs for Addition

Let **a** and **b** be positive integers:

1) Negative plus negative equals negative.

$$-a + (-b) = -(a + b)$$

2) Positive plus negative equals positive if **a > b.**

$$a + (-b) = a - b$$

3) Positive plus negative equals negative if **a < b.**

$$a + (-b) = -(b - a)$$

The number line is a more abstract model for illustrating the addition of positive and negative numbers. Refer to your textbook.

Subtraction - The *take-away model* can be used for subtraction of integers. For example: –5 – (–2) can be illustrated by representing –5 by 5 red chips and then taking away 2 red chips.

Adding Opposites

One common method of subtracting an integer is to add its opposite. Suppose we are given 3 – 5. We can represent three by five black chips and two red chips. Then five black chips can be taken away.

If, instead of removing 5 black chips, we put in five red chips, the final set will still represent –2. In other words, *putting in* 5 *red chips* has the same effect as *taking away* 5 *black chips*.

This suggests that subtracting 5 is the same as adding its opposite, –5. This approach to subtraction is called **adding opposites**. It enables us to compute the difference of any two integers by computing a sum.

Subtraction of Integers For any two integers **a** and **b**, **a** – **b** is the sum of **a** plus the opposite of **b**, $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$.

Math 205 Section 5.1 Integers

Problem Opener: Keeping the single-digit numbers from 1 to 9 in order,

1 2 3 4 5 6 7 8 9

and inserting plus and/or minus signs, we can obtain a sum of 100 in several ways.
For example, $1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 = 100$. Find another way.

The need for negative whole numbers ($-1, -2, -3, \ldots$) originated over 2000 years ago. As trading became more common, whole numbers were needed for two distinctly different uses: to indicate *credits* (or *gains)* and to indicate *debts* (or *losses*). Conventions were developed to permit the use of whole numbers in both cases. About 200 B.C. the Chinese were computing credits with red rods and debts with black rods. Similarly, in their writing they used red numerals and black numerals.

Today it is customary to reverse the color scheme used by the Chinese. Banks often use red numerals to represent amounts below zero ("in the red" is negative). Black numerals are used to represent accounts above zero ("in the black" is positive).

Positive and Negative Integers
The whole numbers, $0, 1, 2, 3, 4, \ldots,$ together with the negatives of the whole numbers, $-1, -2, -3, -4, \ldots,$ are called **integers.**

The concept of positive and negative numbers is useful whenever we wish to count on both sides of a fixed point of reference.

Example 1: Credits and Debits
One common example of opposites is *credits*, which are represented by positive numbers, and *debts* or *deficits,*
which are represented by negative numbers.

Example 2: Temperature
Measuring temperature is another familiar use for positive and negative numbers. The fixed reference point on
the Celsius thermometer is 0, the temperature at which water freezes. On the Fahrenheit scale, water freezes at 32. On both scales, temperatures *above* zero are *positive* and those *below* zero are *negative.*

Example 3: Sports
In several sports there are special reference points from which it is convenient to measure with positive and negative numbers. In golf this reference point is *par*, and a score of – 4 represents four strokes *below par*. In football, the yard line at which play begins is the reference point, and a *loss of yardage* is referred to as *negative yardage.*

Example 4: Time
Scientists often find it convenient to designate a given time as *zero time* and then refer to the *time before* and *time after* as being negative and positive, respectively. This practice is followed in the launching of rockets. If the time with respect to blastoff is –15 minutes, then it is 15 minutes before the launch.

Example 5: Altitude
Sea level is the common reference point for measuring altitudes. Charts and maps that label altitudes below and above sea level use negative and positive numbers.