

MATH 615 Section 4.6 Homework Key

(21) $V = \pi r^2 h$

When $r=3$, $V = \pi(3)^2 h = 9\pi h$

Find $\frac{dV}{dt}$ when $h=4$ ft and $\frac{dh}{dt} = 0.2$ ft/sec.

$V = 9\pi h$

$\frac{d}{dt}[V] = 9\pi \frac{d}{dt}[h]$

$\frac{dV}{dt} = 9\pi \frac{dh}{dt}$

$\frac{dV}{dt} = 9\pi(0.2)$

$\frac{dV}{dt} = 1.8\pi$ ft³/sec

$\frac{dV}{dt} \approx 5.655$ ft³/sec

The volume of the gasoline is increasing at the rate of 5.655 ft³/sec.

(33) $V = \pi r^2 L$

Find $\frac{dr}{dt}$ when $\frac{dL}{dt} = 0.1$ cm/sec and $L=5$ cm
 $r=1$ cm and $\frac{dV}{dt} = 0$ cm³/sec

$V = \pi r^2 L$

$\frac{d}{dt}[V] = \pi \frac{d}{dt}[r^2 L]$

$\frac{dV}{dt} = \pi \left(2r \frac{dr}{dt} L + \frac{dL}{dt} r^2 \right)$

$\frac{dV}{dt} = 2\pi r L \frac{dr}{dt} + \pi r^2 \frac{dL}{dt}$

$0 = 2\pi(1)(5) \frac{dr}{dt} + \pi(1)^2(0.1)$

$0 = 10\pi \frac{dr}{dt} + 0.1\pi$

$-0.1\pi = 10\pi \frac{dr}{dt}$

$-0.01 = \frac{dr}{dt}$

$\frac{dr}{dt} = -0.01$ cm/sec

The radius is decreasing at 0.01 cm/sec.

(31) $V = \frac{4}{3}\pi r^3$

Find $\frac{dV}{dt}$ when $\frac{dr}{dt} = -0.2$ cm/hr and $r=15$ cm.

$V = \frac{4}{3}\pi r^3$

$\frac{d}{dt}[V] = \frac{4}{3}\pi \frac{d}{dt}[r^3]$

$\frac{dV}{dt} = \frac{4}{3}\pi \left(3r^2 \frac{dr}{dt} \right)$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(15)(-0.2)$

$\frac{dV}{dt} = -180\pi$ cm³/hr

$\frac{dV}{dt} \approx -565.487$ cm³/hr

The volume of the snowball is decreasing at a rate of 565.487 cm³/hr.