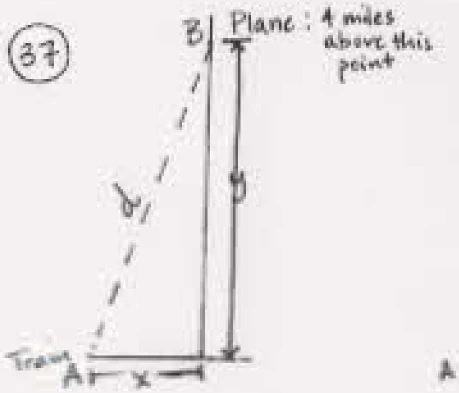
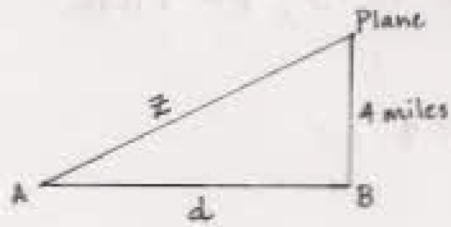
(37)



(View from air)



(Vertical view)

$$d^2 = x^2 + y^2$$

$$z^2 = d^2 + 4^2$$
$$z^2 = d^2 + 16$$
$$z^2 = x^2 + y^2 + 16$$

Find $\frac{dz}{dt}$ when $\frac{dx}{dt} = 80$ mph, $x = 1$ mi, $\frac{dy}{dt} = 500$ mph, and $y = 5$ mi. $\left(z^2 = 1^2 + 5^2 + 16 = 42 \Rightarrow z = \sqrt{42}\right)$

$$\frac{d}{dt}\left[z^2\right] = \frac{d}{dt}\left[x^2\right] + \frac{d}{dt}\left[y^2\right] + \frac{d}{dt}\left[16\right]$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 0$$

$$2(\sqrt{42})\frac{dz}{dt} = 2(1)(80) + 2(5)(500)$$

$$2\sqrt{42}\,\frac{dz}{dt} = 160 + 5000$$

$$\frac{dz}{dt} = \frac{5160}{2\sqrt{42}}$$

$$\frac{dz}{dt} = \frac{2580}{\sqrt{42}} \text{ mph}$$

$$\boxed{\frac{dz}{dt} \approx 398.103 \text{ mph}}$$

The distance between the train and plane is increasing at a rate of 398.103 mph.

(45) (A) height of elevator from the ground : $h(t) = 300 - 30t, 0 \le t \le 10$

(B)
$$\tan\theta = \frac{h(t) - 100}{150}$$

$$\tan\theta = \frac{(300-30t) - 100}{150}$$

$$\tan\theta = \frac{200-30t}{150}$$

$$\Rightarrow \theta = \arctan\left(\frac{200-30t}{150}\right)$$

$$\frac{d}{dt}\left[\theta\right] = \frac{d}{dt}\left[\arctan\left(\frac{200-30t}{150}\right)\right]$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{200-30t}{150}\right)^2}\cdot\left(\frac{-30}{150}\right)$$

$$\frac{d\theta}{dt} = \frac{1}{1+\frac{(200-30t)^2}{150^2}}\cdot -\frac{1}{5}$$

$$\frac{d\theta}{dt} = \frac{1}{\frac{150^2+(200-30t)^2}{150^2}}\cdot -\frac{1}{5}$$

$$\frac{d\theta}{dt} = \frac{150^2}{150^2+(200-30t)^2}\cdot -\frac{1}{5}$$

$$\frac{d\theta}{dt} = -\frac{1}{5}\left(\frac{150^2}{150^2+(200-30t)^2}\right)$$

* $\frac{d\theta}{dt}$ is always negative which is reasonable since θ decreases as the elevator descends.

(C) To find where θ changes (decreases) the fastest, find when $\frac{d\theta}{dt}$ has the largest magnitude. This will occur when the denominator is smallest or when

$$200 - 30t = 0$$
$$-30t = -200$$
$$t = \frac{200}{30}$$
$$t = \frac{20}{3} \text{ seconds}$$

$$h\left(\frac{20}{3}\right) = 300 - 30\left(\frac{20}{3}\right)$$
$$h\left(\frac{20}{3}\right) = 300 - 200$$
$$h\left(\frac{20}{3}\right) = 100 \text{ feet}$$

The elevator appears to be moving fastest when the height is 100 feet.