MATH 205 Section 1.1 Introduction to Problem Solving

”Learning to solve problems is the principal reason for studying mathematics.” (National Council of Supervisors of Mathematics, *Essential Mathematics for the* 21*st Century*, Minneapolis, MN: Essential Mathematics Task Force, 1988)

Definition: A problem exists when there is a situation you want to resolve but no solution is readily apparent. Problem solving is the process by which the situation is resolved.

George Polya, one of the foremost twentieth-century mathematicians, devoted much of his teaching to helping students become better problem solvers. In his book, *How to Solve It*, Polya outlines a four-step process for solving problems:

- **Step 1: Understanding the Problem** - Recognize what is being asked. Become better acquainted with the problem and work toward a clearer understanding of it before progressing toward a solution.
- **Step 2: Devising a Plan** - Respond to what is asked. Most interesting problems do not have obvious solutions. Experience and practice are the best teachers for devising plans.
- **Step 3: Carrying out the Plan** - Develop the result of the response. The plan gives a general outline of direction. Write down your thinking so your steps can be retraced. If you get stuck, it is sometimes better to put aside the problem and return to it later.
- **Step 4: Looking Back** - Check your work. Verify that your results and then interpret them in relation to the original problem.

Polya’s problem-solving steps will be used throughout the text. The purpose of this section is to help you become familiar with the four-step process and to acquaint you with some of the common strategies for solving problems:

1. **Making a Drawing** - One of the most helpful strategies for understanding a problem and obtaining ideas for a solution is to draw sketches and diagrams.
2. **Guessing and Checking** - Many problems can be better understood and even solved by trial-and-error procedures. Even if guessing doesn’t produce the correct answer, you may increase your understanding of the problem and obtain an idea for solving it. This method is especially appropriate for elementary school children because it puts many problems within their reach.
3. **Making a Table** - A problem can sometimes be solved by listing some or all of the possibilities. A table is often convenient for organizing such a list.
4. **Using a Model** - Models are important aids for visualizing a problem and suggesting a solution.
5. **Working Backward** - Working backward can be a tricky problem solving strategy. It can be used when we aren’t given information that would fit in the beginning of a problem.

Let’s solve problems using one or more of the strategies listed above. Utilize some or all of Polya’s four step process to solve each problem.

Problem 1:

(A) Five people enter a racquetball tournament in which each person must play every other person exactly once. Determine the total number of games that will be played.

(B) There are 560 third- and fourth-grade students at King Elementary School. If there are 80 more

third-graders than fourth-graders, how many third-graders are there are there in the school?

Problem 2:

(A) When two numbers are multiplied, their product is 759, but when one is subtracted from the other, their difference is 10. What are these two numbers?

(B) Carmela opened her piggy bank and found she had $15.30. If she had only nickels, dimes, quarters, and half-dollars and an equal number of coins of each kind, how many coins did she have in all?

Problem 3:

(A) Evan had some pennies, nickels, dimes, and quarters in his pocket. When he reached into his pocket and pulled out some change, he had less than 10 coins whose value was 42 cents. What are all the possibilities for the coins Evan had in his hand?

(B) Sasha and Francisco were selling lemonade for 25 cents per half cup and 50 cents per full cup. At the end of the day they had collected $15.00 and had used 37 cups. How many full cups and how many half cups did they sell?

Problem 4:

What is the smallest number of different colors of tile needed to form a 4×4 square so that no tile touches another of the same color along an entire edge?

Problem 5:

Juan and Carlos both have $510 in their savings accounts now. They opened their accounts on the same day, at which time Juan started with $70 more than Carlos. From then on Juan added $10 to his account each week, and Carlos put in $20 each week. How much money did Juan deposit when he initially opened his account?