Math 205 **Section 1.2 Patterns and Problem Solving**

Patterns play a major role in the solution of problems in all areas of life. Find a pattern is such a useful problem-solving strategy that some have called it the art of mathematics. To find patterns, we need to compare and contrast. We must compare to find features that remain constant and contrast to find those that are changing. There are number patterns, geometric patterns, word patterns, and letter patterns, to name a few.

Problem 1: Try finding a pattern in each of the following sequences, and write the possible next three terms.
(A) 1, 2, 4, . . . (B) Al, Bev, Carl, Donna, . . . (C) 1, 4, 9, 17, . . .
(D) 32, 16, 8, 4, 2, . . . (E) 1, 1, 1, 2, 2, 4, 3, 3, 9, . . . (F) $-1, 2, 7, 14, 23, \ldots$

Number patterns have fascinated people since the beginning of recorded history. One of the earliest patterns to be recognized led to the distinction between even numbers and odd numbers.

Problem 2: Find a pattern that might explain the numbering of rows as 0, 1, 2, 3, etc.

Row 0					1					
Row 1				1		1				
Row 2			1		2		1			
Row 3		1		3		3		1		
Row 4	1		4		6		4		1	

Sequences of numbers are often generated by patterns. If each new number is obtained from the previous number in a sequence by adding a selected number throughout, the selected number is called the **common difference**, and the sequence is called an **arithmetic sequence**. If each new number is obtained by multiplying the previous number by a selected number, then the sequence is a **geometric sequence**, and the selected number is called the **common ratio**.

Problem 3: Try finding a pattern in each of the following sequences, and write the possible next three terms. Determine whether the sequence is an arithmetic sequence or a geometric sequence.
(A) 8, –1, –10, –19, . . . (B) $-1, -\frac{1}{3}, -\frac{1}{9}, -\frac{1}{27}, \ldots$ (C) 2, 4, 6, 8, . . .

Often sequences of numbers don't appear to have a pattern. However, sometimes number patterns can be found by looking at the differences between consecutive terms. This approach is called the method of **finite differences.**

Problem 4: Use the method of finite differences to determine the possible next term in each sequence.
(A) 14, 22, 32, 44, ... (B) 5, 15, 37, 77, 141, ...

Inductive Reasoning

The process of forming conclusions on the basis of patterns, observations, examples, or experiments is called **inductive reasoning**. Identifying patterns is a powerful problem solving strategy. It is also the essence of inductive reasoning. As students explore problem situations appropriate to their grade level, they can often consider or generate a set of specific instances, organize them, and look for a pattern. These, in turn, can lead to conjectures about the problem. Inductive reasoning may be thought of as making an “informed guess.” Although this type of reasoning is important in mathematics, it sometimes leads to incorrect results.

An example that shows a statement to be false is called a **counterexample.** If you have a general statement, test it to see if it is true for a few special cases. You may be able to find a counterexample to show that the statement is not true, or that a conjecture cannot be proved.

Problem 5: For which of the following statements is there a counterexample? If a statement is false, change a condition to produce a true statement.

(A) The product of any three consecutive whole numbers is divisible by 2. (#42-a)

(B) The sum of any two consecutive whole numbers is divisible by 2. (#42-b)