**Math 205  Section 1.3 Problem Solving with Algebra**

Algebra is a powerful tool for representing information and solving problems. Algebra originated in Babylonia and Egypt more than 4000 years ago. At first there were no equations, and words rather than letters were used for variables. The Egyptians used words that have been translated as *heap* and *aha* for unknown quantities in their word problems. Here is a problem from the Rhind Papyrus, written by the Egyptian priest Ahmes about 1650 B.C.:

*Heap and one-seventh of a heap is* 19*. What is a heap?*

Today we would use a letter for the unknown quantity and express the given information in an equation: $x + \frac{1}{7}x = 19$

A letter or symbol that is used to denote an unknown number is called a **variable**. An expression containing algebraic symbols, such as $2x + 3$ or $(4)(7x) - 5$, is called an **algebraic expression.**

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**Problem 1:** (a) A woman's shoe size is given by $3x - 22$, where $x$ is the length of her foot in inches. What is a woman's shoe size for length of 10 inches? 12 inches?

(b) The number of words in a child's vocabulary for children between 20 and 50 months is $60x - 900$, where $x$ is the child's age in months. What is the number of vocabulary words for a child whose age is 22 months? 38 months? 4 years?

(c) A person's maximum heart rate is $220 - x$, where $x$ is the person's age, and the heart rate for aerobic activity should be between $0.7(220 - x)$ and $0.8(220 - x)$. A 22-year-old person's heart rate for aerobic activity should be between what two numbers?

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**Problem 2:** At the Saturday farmers' market, melons cost $1.35 each and coconuts cost $1.55 each. Let *m* represent

the number of melons sold during the day, and let *c* represent the number of coconuts sold. Write an algebraic expression for each of the following.

A) The total number of melons and coconuts sold    B) The cost of all the coconuts sold

C) The total cost of all the melons and coconuts sold

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**Problem 3:** Mr. Dawson purchases some artichokes for 80 cents each and twice as many pineapples for 95 cents each. Altogether he spent $24.30. Let *x* represent the number of artichokes, and write an algebraic expression for each item in parts a through c.

A) The total cost in dollars of the artichokes    B) The number of pineapples    C) The total cost in dollars of the pineapples

D) Write and solve an equation to determine the number of artichokes Mr. Dawson bought.

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**Problem 4:** Determine the number of chips needed to replace each box in order for the scales to balance. Then using *x* to represent the number of chips for each box, write the corresponding equation that represents each scale and solve the equations. See the figures in your textbook on page 50, exercise #6.

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**Solving Inequalities**

Not all algebra problems are solved by equations. We can use inequalities to solve some algebra problems. An **inequality** is a statement that uses one of the following phrases: *is less than* ($<$), *is less than or equal to* ($\leq$), *is greater than* ($>$), *is greater than or equal to* ($\geq$), or is not equal to ($\neq$). To **solve an inequality** means to find all the replacements for the variable that make the inequality true. The replacements that make the inequality true are called **solutions**. Like an equation, an inequality is solved by replacing it by simpler inequalities. Two inequalities that have exactly the same solution are called **equivalent inequalities**.

Equivalent inequalities can be obtained by using the same steps as those for obtaining equivalent equations (performing the same operation on both sides and replacing an expression by an equivalent expression), with one exception: Multiplying or dividing both sides of an inequality by a negative number **reverses the inequality**.

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**Problem 5:** Solve each inequality and illustrate the solution by using a number line.

(A) $3x + 5 < x + 17$ (B) $3(7 - 2x) \geq 36$

(C) Determine the number of chips for each box that will keep the scales tipped as shown. Then using *x* for a variable, write the corresponding inequality for each scale and solve the inequality. See the figures in your textbook on page 50, exercise #12.

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**Problem 6:** Solve by writing an inequality with a variable and then solve the inequality.

The length of the first side of a triangle is a whole number greater than 3. The second side is 3 inches longer than the first, and the third side is 3 inches longer than the second. How many such triangles have perimeters less than 36 inches?