Math 205 **Section 2.3 Introduction to Deductive Reasoning**

There are two main types of reasoning–*inductive* and *deductive,* and both are common in forming conclusions in our everyday activities.

Example of Inductive Reasoning (Review from Chapter 1)

Observation: An employer notices that an emplyee has always been thirty minutes early for work on Wednesdays.

Conclusion: The employee will be thirty minutes early for work the next Wednesday.

This conclusion may or may not be true, but we cannot be sure from the given information. Conclusions based on inductive reasoning are sometimes called *informed guesses.*

Deductive reasoning, however, is the process of forming conclusions from one or more given statements.

Example of Deductive Reasoning

Given Statements: 1) The sum of two numbers is 314. 2) One of the numbers is 132.

Conclusion: The other number is 182.

With *inductive reasoning* we form a conclusion that is probable or likely, and with *deductive reasoning* we form a conclusion based on given statements.

The *Curriculum and Evaluation Standards for School Mathematics* discusses the importance of both types of reasoning:

> Both inductive and deductive reasoning come into play as students make conjectures and seek to explain why they are valid. Whether encouraged by technology or by challenging mathematical situations posed in the classroom, this freedom to explore, conjecture, validate, and to convince others is critical to the development of mathematical reasoning in the middle grades.

Venn Diagrams

Examples of Deductive Reasoning Using Venn Diagrams

Problem 1: **Premises**

1) All teachers are smart. 2) All smart people graduate from college.

Conclusion:

With a diagram, we can illustrate that the teachers are a subset of the smart people and the smart people are a subset of those who graduate college. So teachers are a subset of those who graduate college. (With deductive reasoning, it is possible to obtain conclusions from given statements without necessarily having an understanding of the subject matter.)

Problem 2: **Premises**

1) All salamanders are amphibians.

2) Animals that develop an amnion are not amphibians.

Conclusion:

(*An amnion is the innermost of the embryonic or fetal membranes of reptiles, birds, and mammals; the sac in which the embryo is suspended.* Source: www.dictionary.com)

Problem 3: **Premises**

1) All logicians are mathematicians.
2) Some philosophers are not mathematicians.

Conclusion:

Problem 4: **Premises**

1) Some members the Strategic Planning Committee are Democrats.
2) Some Democrats are on the Budget Committee.

Conclusion:

Conditional Statements

A **conditional statement** is a statement of the form "If..., then..." The "if" part is called the **hypothesis**, and the "then" part is called the **conclusion**. Many statements that are not in if-then form can be rewritten as conditional statements.

Examples:

1) If a number is less than 3, then it is less than 8.
2) If tomorrow is Thursday, then today is Wednesday.
3) If Bill wins the lottery, then he will buy a new house.
4) Every public beach must have a lifeguard.
5) All courses completed with a grade of C will not count for graduate credit.

Every conditional statement "if p, then q" has three related conditional statements that can be obtained by negating and/or interchanging the *if* part and the *then* part. The new statements each have special names that show their relationship to the original statement.

Statement	If p, then q.
Converse	If q, then p.
Inverse	If not p, then not q.
Contrapositive	If not q, then not p.

Problem 5: Using each of the conditional statements given above, write the related inverse, converse, and contrapositive. For statements 4 and 5, first write the conditional statement.

Reasoning with Conditional Statements

Venn diagrams can be used to show whether conclusions drawn from conditional statements are valid or invalid.

Problem 6: Determine whether the conclusion is valid or invalid.

Premises: 1) If a person challenges a creditor's report, then the credit bureau will conduct an investigation for that person.

2) Ronald C. Whitney challenged a creditor's report.

Conclusion: The credit bureau will conduct an investigation for Ronald C. Whitney.

When a conditional statement is given and the *if part* is satisfied, the *then part* will logically follow. This principle is known as the **law of detachment.**

LAW OF DETACHMENT **Premises**

1) If p, then q.

2) p

Conclusion

q (valid)

Problem 7: Determine if the conclusion is valid or invalid.

Premises: 1) If the temperature drops below $65^\circ F$, then the heat rheostat is activated.

2) The heat rheostat was not activated on Wednesday.

Conclusion: The temperature did not drop below 65° on Wednesday.

When a conditional statement is given and the negation of the *then part* is satisfied, the negation of the if *part* will logically follow. This principle is known as the **law of contraposition.**

LAW OF CONTRAPOSITION **Premises**

1) If p, then q.

2) NOT q

Conclusion

NOT p (valid)

Problem 8: (p. 118 #32)

Janet Davis, Sally Adams, Collette Eaton, and Jeff Clark have occupations of architect, carpenter, diver, and engineer, but not necessarily in that order. You are told: (1) The first letters of a person's last name and occupation are different. (2) Jeff and the engineer go sailing together. (3) Janet lives in the same neighborhood as the carpenter and engineer. Determine each person's occupation.

	Architect	Carpenter	Diver	Engineer
Janet Davis				
Sally Adams				
Collette Eaton				
Jeff Clark				