Math 205 Section 2.2 Functions and Graphs

The Curriculum and Evaluation Standards for School Mathematics comments on the importance of functions:

> One of the central themes of mathematics is the study of patterns and functions. This study requires students to recognize, describe, and generalize patterns and build mathematical models to predict the behavior of real-world phenomena that exhibit the observed pattern. The widespread occurrence of regular and chaotic pattern behavior makes the study of patterns and functions important. (Reston, VA: National Council of Teachers of Mathematics, 1989, p. 98).

Functions arise whenever one quantity depends on another. Consider the following three situations:

1. The area A of a circle depends on the radius r of the circle. The rule that connects r and A is given by the equation $A = \pi r^2$. With each positive number r there is associated one value of A, and we say that A is a **function** of r.
2. The human population of the world P depends on the time t. The table below give estimates of the world population $P(t)$ at time t, for certain years. For instance, $P(1950) \approx 2,520,000,000$. But for each value of the time t there is a corresponding value of P, and we say that P is a function of t.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	1996
Population (millions)	1650	1750	1860	2070	2300	2520	3020	3700	4450	5300	5770

3. The cost C of mailing a first-class letter depends on the weight w of the letter. Although there is no simple formula that connects w and C, the post office has a rule for determining C when w is known.

Each of these examples describes a rule whereby, given a number ($r, t,$ or w), another number ($A, P,$ or C) is assigned. In each case we say that the second number is a function of the first number.

There are four ways to represent a function:

- Verbally (by a description in words such as in Examples 1 and 2 above)
- Numerically (by a table of values such as in the table in Example 2 above)
- Visually (by a graph – If we graph the graph the information given in the table in Example 2 above, we would represent the data graphically.)
- Algebraically (by an explicit formula as in Example 1 above)

If a single function can be represented in all four ways, it is often useful to go from one representation to another to gain additional insight into the function.

So, what is a function? A **function** is *two sets* and a *rule* that assigns to each element of the first set to an element of the second set so that no element of the first set is assigned to no more than one element of the second set. The first set is called the **domain**, and the second set is called the **range**. The rule for a function is often defined by an algebraic formula.. It is customary to refer to an arbitrary element of the domain by a variable, such as x, and the corresponding element of the range by $f(x)$ [or $g(x), s(x), h(x),$ etc.]. The symbol is read as "f of x" [or "g of x", "s of x", "h of x"].

NOTE: $f(x)$ does not mean f times x.

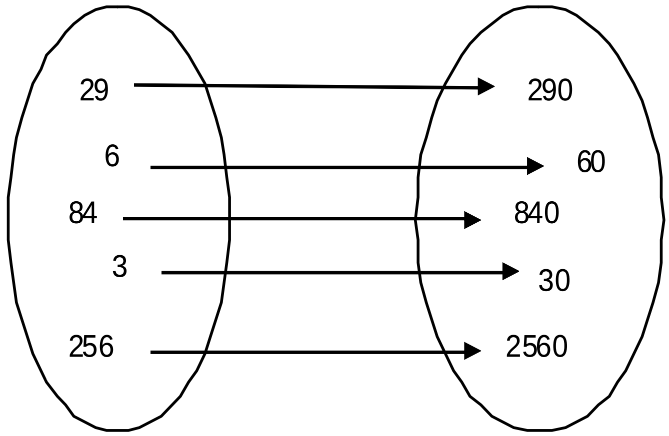
Problem 1: Determine which rules are functions. If the rule is not a function, explain why.

A) People assigned to their ages
B) Biological mother assigned to her child
C) Child assigned to biological mother
D) Vehicle assigned to vehicle identification number
E) Birthday assigned to person
F) Each person assigned to a person who is older.

Problem 2: Give an example of a function from everyday life and explain why your example is a function.

One visual method of illustrating the assignment of elements from the domain to their corresponding elements in the range is with **arrow diagrams**.

Problem 3: Describe a rule for assigning each element of the domain to an element of the range for the arrow diagram.

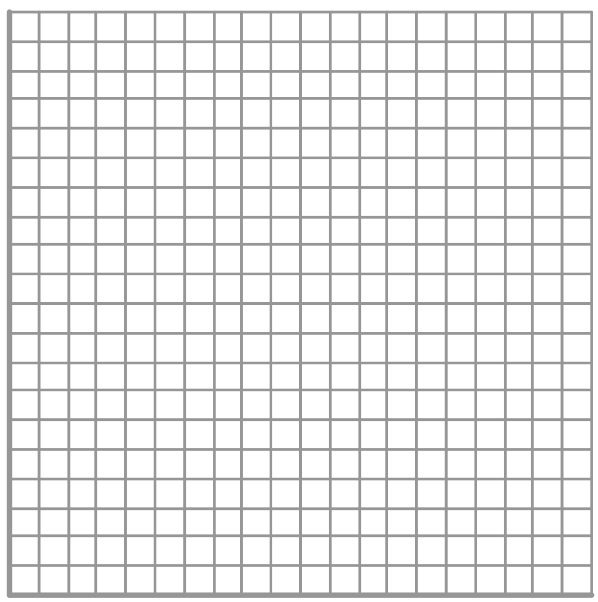


A) Write an algebraic rule for $g(x)$ that describes what each x in the domain corresponds to in the range.

B) Complete the table below.

x	1	2	3	4	5	6	7	8	9	10
$g(x)$										

C) Using a rectangular grid, plot the points whose coordinates are given in the table in part (B) and then graph g.



Problem 4: Write an algebraic rule for each of the following functions, where the domain is all whole numbers and *x* represents an element of the domain.

A) $g(x)$ is an element of the range, and each element in the domain is assigned to eight less than three times its value.

B) $f(x)$ is an element of the range, and each element in the domain is assigned to four times its value.

C) $h(x)$ is an element of the range, and each element in the domain is five more than half its value.

D) Evaluate $g(2), f(5),$ and $h(150)$.

Rectangular Coordinates

Graphs provide a visual method for illustrating functions. A **horizontal axis**, called the *x-axis*, is used for the elements of the domain. A **vertical axis**, called the *y-axis,* is used for the elements of the range.

Each point on a graph is located by two numbers; their order is significant. The first number is called the *x-coordinate* and indicates the distance to the right or left of the vertical axis. The second number is called the *y-coordinate* and indicates the distance above or below the horizontal axis.

The intersection of the two axes is called the **origin** and has coordinates (0, 0).

This method of locating and describing points is called the **rectangular** (or **Cartesian**) **coordinate system**.

Problem 5: Plot the following points in the rectangular coordinate system.

$A(3,-4)$ $B(0,3)$ $C(-1,-5)$ $D(-2,1)$ $E(2,2)$

Linear Functions and Slope

The concept of slope occurs in many applications of mathematics. The slope of a line is defined as the steepness of a line. Two points on a line are selected, and the **slope** of the line connecting these points is the difference between the two y coordinates (the **rise**) divided by the difference between the two x coordinates (the **run**).

Problem 6: Find the rise and run for each pair of points and determine the slope of the line, if it exists.

(See textbook, page 83.)

In general, lines that extend from lower left to upper right have a **positive slope**, and lines that extend from upper left to lower right have a **negative slope**.

The y coordinate of the point at which the graph of a line crosses the vertical axis is called the **y intercept**. The x coordinate of the point at which the graph of a line crosses the horizontal axis is called the **x intercept**.

Problem 7: Find the x intercept and y intercept for each line given by the equation.

(A) $3x - 2y = -12$ (B) $y = \frac{2}{5}x + 30$

Functions whose graphs are lines that are not parallel to the vertical axis are called **linear functions**. When the equation of the line is written in $y = mx + b$ form, it is said to be in **slope-intercept form** because the slope m of the line and the y intercept b can be easily determined from the equation.

Problem 8: Graph each linear equation.

(A) $2x + y = -2$ (B) $y = -\frac{1}{2}x + 3$ (C) $y = \frac{2}{5}x - 2$