Math 205 - Numeration Systems

BASE TEN

As soon as people were accustomed to counting by the fingers on one hand, it became natural to use the fingers on both hands to group by 10s. In most numeration systems today, grouping is done by 10s. When grouping is done by 10s, the system is called a **base-ten numeration system.**

Hindu-Arabic Numeration

Much of the world now uses the Hindu-Arabic numeration system. This positional numeration system was named for the Hindus, who invented it, and the Arabs, who transmitted it to Europe. It is a base-ten numeration system in which place value is determined by the position of the **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each digit in a numeral has a name that indicates its position.

Example 1: Here are the names and values of the digits in 84,607.

Ten thousands digit		Thousands digit		Hundreds digit		Tens digit		Units (ones) digit
8		4		6		0		7
$8 \times 10,000$	+	$4 \times 1,000$	+	6×100	+	0×10	+	7×1

When we write a number as the sum of the numbers represented by each digit in its numeral, we are writing the number in **expanded form**. Another common method of writing a number in expanded form is to write the powers of the base using exponents. For example, $8 \times 10^4 + 4 \times 10^3 + 6 \times 10^2 + 0 \times 10 + 7$.

Positional Numeration System

In a positional numeration system, a number is selected for a base and basic symbols are adopted for 0, 1, 2,… up to one less than the base. (In our numeration system these basic symbols are the 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Whole numbers are represented in a positional numeration system by writing one or more basic symbols side by side with their positions indicating increasing powers of the base.

The word ”and” is not used in reading a whole number. For example, the number 23,478,506,042,319 is read ”twenty-three trillion, four hundred seventy-eight billion, five hundred six million, forty-two thousand, three hundred nineteen.

Models for Numeration

NCTM’s K–4 Standard, *Number Sense and Numeration*, says that place value is a critical step in the development of children’s understanding of number concepts.

Place-value meanings grow out of grouping experiences. Therefore counting knowledge should be integrated with meanings based on grouping.

Base-Ten Pieces

The powers of 10 are represented by objects called **units, longs, and flats**. 10 units form a long; 10 longs form a flat; and 10 flats end to end form a long-flat.

Example 2: Sketch/describe base-ten pieces to represent 426.

Example 3: Suppose you have 8 flats, 13 longs, and 23 units. Sketch/describe the minimum number of base-ten pieces needed to replace the old collection. Then determine the base-ten number represented by the new collection.

BASE FIVE

Base-Five Numeration

The base-five pieces are models for powers of 5, and there are pieces called **units, longs,** and **flats.** Five units form a long; five longs form a flat; five flats end to end form a long-flat. The digits in base five are 0, 1, 2, 3, and 4. A number in base-five numeration is written with a subscript of five. For example, 115 is written as 430_{five}. We do not write the subscript to show that a number is written in base-ten numeration since base ten is the standard base.

Example 4: Sketch/describe the minimum number of base-five pieces to represent the following number of units.
A) 43 units B) 172 units C) 632 units

Example 5: Express the units in Example 4 as a number in base five.

Example 6: What digits are needed for base four and what would the base-four pieces look like?
Positional numeration is used to write numbers in various bases by writing the numbers of long-flats, flats, long, and units from left to right, just as we do in base ten.

Example 7:

1) Base Two: Digits are 0 and 1.
Example: (A) Convert 1001_{two} to base ten. (B) Convert 143 to base two.

2) Base Three: Digits are 0, 1, and 2.
Example: (A) Convert 102_{three} to base ten. (B) Convert 413 to base three.

3) Base Seven: Digits are 0, 1, 2, 3, 4, 5, and 6.
Example: (A) Convert 13425_{seven} to base ten. (B) Convert 357 to base seven.

4) Base Twelve: Ten in base twelve represents 1 long and 0 units, which equals twelve units; and eleven in base twelve represents 1 long and 1 unit, which equals 13 units. One solution is to let T represent the number 10 and E represent the number eleven. Digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, and E.
The first few numerals in base twelve are 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E, 10, 11, 12, 13, …, where 12 represents 1 long and 2 units (fourteen units), etc. In base-twelve positional numeration, 3 flats, 2 longs, and 8 units are written 328_{twelve}.