Math 205 Section 2.1 Sets and Venn Diagrams

The human mind likes to create collections. Instead of seeing a group of five stars as five separate items, people tend to see them as one group of stars. The mind tries to find order and patterns. In mathematics this tendency to create collections is represented with the idea of a **set**. A **set** is a collection of objects. The objects belonging to the set are called the **elements,** or **members**, of the set.

In every problem there is either a stated or implied **universe of discourse**. The universe of discourse includes all things under discussion at a given time. In the mathematical theory sets, the universe of discourse is called the **universe set.** The letter U is typically used for the universal set. The universal set may change from problem to problem.

Problem 1: Each of the following sets is described in set-builder notation. Write each of the sets by listing its elements.
(A) $A = \{2k+1 | k = 3, 4, 5\}$
(B) $B = \{a^2 + b^2 | a = 2 \text{ or } 3, \text{ and } b = 2, 3, 4\}$
(C) $\{x | x \text{ is a counting number between 16 and 17}\}$

Problem 2: Write the following sets using set-builder notation.
(A) $\{2, 4, 6, 8, 10, \ldots\}$
(B) $\{1, 3, 5, 7, \ldots\}$

A set containing no elements, such as the set in Problem 1(C), is called the **empty set**, or **null set**. The symbol $\emptyset$ often is used to denote the empty set, so $\emptyset$ and $\{\quad\}$ have the same meaning. We do not denote the empty set with the symbol $\{\emptyset\}$ since this notation represents a set with one element (that element being the empty set).

Relationships Between Sets

- When two sets have no elements in common, we say the two sets are **disjoint.**
- If every element of a set A is also an element of a set B, then set A is a **subset** of B. This relationship is written $A \subseteq B$. If we know that $A \subseteq B$ and that one or more elements of B are not in A, then A is called a **proper subset** of B and we write $A \subset B$.
- If A is a subset of B and B is a subset of A, then both sets have exactly the same elements and they are **equal**. This relationship is written $A = B$. In this case A and B are just different letters naming the same set.
- On the other hand, two sets are **equivalent** if they contain the same number of elements (but not necessarily the same elements).
- A set is **finite** if it is empty or if it can be put into a one-to-one correspondence with set $\{1, 2, 3, \ldots, n\}$, where n is a whole number. A set is **infinite** if it is not finite.

Problem 3: Write $\subseteq$ or $\not\subseteq$ to make a true statement.
(A) $\{3, 4, 5, 6\}$_____$\{3, 4, 5, 6, 8\}$
(B) $\{1, 2, 3\}$______$\{2, 4, 6, 8\}$
(C) $\{5, 6, 7, 8\}$_______$\{5, 6, 7, 8\}$

Set Operations

Let A and B be any sets, with U the universal set.

- The **complement of A**, written A', is $A' = \left\{x | x \in U \text{ and } x \notin A\right\}$.
- The **intersection of A and B** is $A \cap B = \left\{x | x \in A \text{ and } x \in B\right\}$.
- The **union of A and B** is $A \cup B = \left\{x | x \in A \text{ or } x \in B\right\}$.

Problem 4: Let $\begin{aligned} U &= \{a,b,c,d,e,f,n\} \\ A &= \{a,b,c,d\} \\ B &= \{b,d,f\} \\ C &= \{a,c,f,n\} \end{aligned}$. Find each set.

(A) $A' \cap B$ (B) $A \cap B$ (C) $B' \cup C'$

(D) $A \cap (B \cup C')$ (E) $(A' \cup C') \cap B'$ (F) $(A \cap B) \cup C$

In most areas of mathematics, our reasoning can be aided and clarified by utilizing various kinds of drawings and diagrams. In set theory, we commonly use **Venn Diagrams**, developed by the logician John Venn (1834-1923). In these diagrams, the universal set is represented by a rectangle, and other sets of interest within the universal set are depicted by oval regions, or sometimes by circles or other shapes.

Problem 5: In a survey of 110 college freshmen that investigated their high school backgrounds, the following information was gathered:

25 took physics	8 took biology and mathematics
45 took biology	6 took physics and biology
48 took mathematics	5 took all three subjects
10 took physics and mathematics	

Using the survey data on student preferences for performers, draw a Venn diagram and answer the following questions.

(A) How many students took biology, but neither physics nor mathematics?

(B) How many students took physics, biology, or mathematics?

(C) How many students took none of the three subjects?

Problem 6: A survey of 130 aides at the United Nations revealed that 65 could speak English, 80 could speak Spanish, and 55 could speak both English and Spanish.

A) Draw a Venn diagram representing the 130 aides. Use E to represent English-speaking and S to represent French-speaking aides.

B) How many aides are in $E \cap S$?

C) How many aides are in $E \cup S$?

D) How many aides are in $E \cap S'$?

Problem 7: Draw either a two-circle or three-circle Venn diagram and shade the region represented by the set.

A) $A \cap B'$ (B) $(A \cap B) \cup C$ (C) $(A \cup B)'$